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Competing risks models

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Competing Risks Models

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Abstract

A competing risks model is a model for multiple durations that start at the same point of time for a given subject, where the subject is observed until the first duration is completed and one also observes which of the durations is completed first. This article gives an overview of the main issues in the empirical econometric analysis of competing risks models. The central problem is the non-identification of dependent competing risks models. Models with regressors can overcome this problem, but it is advisable to include additional data. Alternatively, effects of interest can be bounded.

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Competing Risks Models

A competing risks model is a model for multiple durations that start at the same point of time for a given subject, where the subject is observed until the first duration is completed and one also observes which of the multiple durations is completed first.

The term 'competing risks' originates from the interpretation that a subject faces different risks i of leaving the state it is in, each risk giving rise to its own exit destination which can also be denoted by i. One may then define random variables T_i describing the duration until risk i is materialized. Only the smallest of all these durations $Y := \min_i T_i$ and the corresponding actual exit destination, which can be expressed as $Z := \arg\min_i T_i$, are observed. The other durations are censored in the sense that all is known is that their realizations exceed Y. Often those other durations are latent or counterfactual, for example if T_i denotes the time until death due to cause i.

In economics, the most common application concerns individual unemployment durations. One may envisage two durations for each individual: one until a transition into employment occurs and one until a transition into non-participation occurs. We only observe one transition, namely the one occurring first. Other applications include the duration of treatments, where the exit destinations are relapse and recovery, and the duration of marriage, where one risk is divorce and the other is death of one of the spouses. More in general, the duration until an event of interest may be right-censored due to the occurrence of another event, or due to the data sampling design. The duration until the censoring is then one of the variables T_i.

Sometimes one is only interested in the distribution of Y. For example, an unemployment insurance agency may only be concerned about the expenses on UI and not in the exit destinations of recipients. In such cases one may employ standard statistical duration analysis for empirical inference with register data on the duration of UI receipt. However, in studies on individual behavior, one is typically interested in one or more of the marginal distributions of the T_i. If these variables are known to be independent then again one may employ standard duration analysis for each of the T_i separately, treating the other variables $T_j (j \neq i)$ as independent right-censoring variables. But often it is not clear whether the T_i are independent. Indeed, economic theory often predicts that they are dependent, in particular if they can be affected by the individual's behavior and individuals are heterogeneous. It may even be sensible from the individual's point of view to use their privately observed exogenous exit rates into destinations j as inputs for the optimal strategy affecting the exit rate into destination $i (i \neq j)$ (see e.g. Van den Berg, 1990). Erroneously assuming independence leads to incorrect inference, and in fact the issue of whether the durations T_i are related is often an important question in its own right.

Unfortunately, the joint distribution of all T_i is not identified from the joint distribution of Y, Z, a result that goes back to Cox (1959). In particular, given any specific joint distribution, there is a joint distribution with independent durations T_i that generates the same distribution of the observable variables Y, Z. In other words, without additional structure, each dependent competing risks model is observationally equivalent to an independent competing risks model. The marginal distributions in the latter can be very different from the true distributions.

Of course, some properties of the joint distribution are identified. To describe these it is useful to introduce the concept of the hazard rate of a continuous duration variable, say W. Formally, the hazard rate at time t is $\theta(t) := \lim_{dt \downarrow 0} \Pr(W \in [t, t+dt))/dt$. Informally, this is the rate at which the duration W is completed at t given that it has not been completed before t. The hazard rate is the basic building block of duration analysis in social sciences because it can be directly related to individual behavior at t. The data on Y, Z allow for identification of the hazard rates of T_i at t *given* that $T \geq t$. These are called the 'crude' hazard rates. If the T_i are independent then these equal the 'net' hazard rates of the marginal distributions of the T_i.

We now turn to a number of approaches that overcome the general non-identification result for competing risks models. In econometrics, one is typically interested in covariate or regressor effects. The main approach has therefore been to specify semi-parametric models that include observed regressors X and unobserved heterogeneity terms V. With a single risk, the most popular duration model is the Mixed Proportional Hazard (MPH) model, which specifies that $\theta(t|X = x, V) = \psi(t) \exp(x'\beta) V$ for some function $\psi(.)$. V is unobserved, and the composition of the survivors changes selectively as time proceeds, so identification from the observable distributions of $T|X$ is non-trivial. However, it holds under the assumptions that $X \perp\!\!\!\perp V$ and $\text{var}(X) > 0$ and some regularity assumptions (see Van den Berg, 2001, for an overview of results). With competing risks, the analogue of the MPH model is the Multivariate MPH (MMPH) model. With two risks,

$$\begin{aligned}\theta_1(t|x,V) &= \psi_1(t)\exp(x'\beta_1)V_1 \qquad \text{and} \\ \theta_2(t|x,V) &= \psi_2(t)\exp(x'\beta_2)V_2.\end{aligned}$$

where $T_1, T_2|X, V$ are assumed independent, so that a dependence of the durations given X is modelled by way of their unobserved determinants V_1 and V_2 being dependent. Many empirical studies have estimated parametric versions of this model, using maximum likelihood estimation.

The semi-parametric model has been shown to be identified, under only slightly stronger conditions than those for the MPH model (Abbring and Van den Berg, 2003). Specifically, $\text{var}(X) > 0$ is strengthened to the condition that the vector X includes two continuous variables with the properties that (i) their joint support contains a non-empty open set in $\mathbb{R}^2$, and (ii) the vectors $\widetilde{\beta_1}, \widetilde{\beta_2}$ of the corresponding elements of β_1 and β_2 form a matrix $(\widetilde{\beta_1}\ \widetilde{\beta_2})$ of full rank. Somewhat loosely, X has two continuous variables that are not perfectly collinear and that act differently on θ_1 and θ_2. Note that with such regressors, one can manipulate $\exp(x'\beta_1)$ while keeping $\exp(x'\beta_2)$ constant. The two terms $\exp(x'\beta_i)$ are identified from the observable crude hazards at $t = 0$ because at $t = 0$ no dynamic selection due to the unobserved heterogeneity has taken place yet. Now suppose one manipulates x in the way described above. If $T_1, T_2|X$ are independent then the observable crude hazard rate of T_2 at $t > 0$, given that $T_1 \geq t$, does not vary along. But if $T_1, T_2|X$ are dependent then this crude hazard rate does vary along, for the following reason. First, changes in $\exp(x'\beta_1)$ affect the distribution of unobserved heterogeneity V_1 among the survivors at t, due to the well-known fact that V_1 and X are dependent conditional on survival $T_1 \geq t > 0$ even though they are independent unconditionally. Secondly, if V_1 and V_2 are dependent this affects the distribution of V_2 among the survivors at t, which in turn affects the observable crude hazard of T_2 at t given that $T_1 \geq t$. In sum, the variation in this crude hazard with $\exp(x'\beta_1)$ for given $\exp(x'\beta_2)$ is informative on the dependence of the durations. An analogous argument holds for the crude hazard rate corresponding to cause $i = 1$.

Note that identification is not based on exclusion restrictions of the sort encountered in instrumental variable analysis, which require a regressor that affects one endogenous variable but not the other. Here, all explanatory variables are allowed to affect both duration variables – they are just not allowed to affect the duration distributions in the same way. Identification with regressors was first established by Heckman and Honoré (1989) who considered a somewhat larger class of models than the MMPH model and accordingly imposed stronger conditions on the support of X.

Although the MPH model is identified from single-risk duration data where we observe a single spell per subject, there is substantial evidence that estimates are sensitive to misspecification of functional forms of model elements (see Van den Berg, 2001, for an overview). This implies that estimates of MMPH models using competing-risks data should also be viewed with caution. It is advisable to include additional data. For example, longitudinal survey data on unemployment durations subject to right-censoring can be augmented with register data or retrospective data not subject to censoring (see e.g. Van den Berg, Lindeboom and Ridder, 1994). More in general, one may resort to 'multiple-spell competing risks' data, meaning data with multiple observations of Y, Z for each subject. For a given subject, such observations can be viewed as multiple independent draws from the subject-specific distribution of Y, Z, assuming that the unobserved heterogeneity terms V_1, V_2 are identical across the spells of the subject. Here, a subject can denote a single physical unit, like an individual, for which we observe two spells in exactly the same state, or it can denote a set of physical units for which we observe one spell each. Multiple-spell data allow for identification under less stringent conditions than single-spell data. Abbring and Van den Berg (2003) showed that such data identify models that allow for full interactions between the elapsed durations t and x in $\theta_i(t|x, V)$, and, indeed, allow the corresponding effects to differ between the first and the second spell. The assumptions on the support of X are similar to above. Fermanian (2003) develops a non-parametric kernel estimator of the Heckman and Honoré (1989) model.

Another approach to deal with non-identification of dependent competing risks models is to determine bounds on the sets of marginal and joint distributions that are compatible with the observable data. Peterson (1976) derived sharp bounds in terms of observable quantities. They are often wide. In case of the marginal distributions of two sub-populations distinguished by a variable X, the bounds associated with the different X may overlap, whether X (monotonically) affects (one of) the marginal distributions or not. With overlap, the causal effects of X cannot even be signed.

Bond and Shaw (2003) combine bounds with regressors. In the case of a single binary regressor, the only substantive assumption made is that there exist increasing functions g and h such that $T_1, T_2|X = 0$ equals $g(T_1), h(T_2)|X = 1$ in distribution. In words, the dependence structure is invariant to the values of the regressors, so the latter only affect the marginal distributions. Specifically, the copula (and therefore Kendall's τ) of the joint distribution is invariant to the value of X. The assumption is satisfied by the above-mentioned competing risks models with regressors. Clearly, by itself the assumption it is insufficient for point identification. The bounds concern the regressor effects on the marginal distributions. If it is assumed that X affects the marginal distributions of T_i in terms of first-order stochastic dominance, the bounds are sufficient to sign the effect of X on at least one of the marginal distributions (so, in case of MMPH models, also on at least one of the individual marginal distributions conditional on V).

We end this article by noting some connections between competing risks models and other models. First, they are related to switching regression models or Roy models. For example, if $T_i|X, V$ in the MMPH model have Weibull distributions then we can write $\log T_i = x_i'\alpha_i + \varepsilon_i (i = 1, 2)$ (e.g. Van den Berg, Lindeboom and Ridder, 1994), where we observe T_i iff $T_i < T_j (j \neq i)$. Secondly, competing risks models are building blocks of multivariate duration models, notably models where one of the durations is always observed (e.g. T_1 captures the moment of a treatment and T_2 is the observed duration outcome of interest).

We have only considered continuous-time duration variables T_i that have different realizations with probability one. Recently, semi-parametric and non-parametric results have been derived for discrete-time or interval-censored competing risks models and models where different risks can be realized simultaneously (see e.g. Bedford and Meilijson, 1997, Van den Berg, Van Lomwel and Van Ours, 2004, Honoré and Lleras-Muney, 2004). The biostatistical literature contains many studies in which specific assumptions are made on the dependence structure of the two durations T_i, enabling inference on the marginal distributions from data on Y, Z (see e.g. Moeschberger and Klein, 1995, for a survey).

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