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The marginal cost of public funds:

A brief guide

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The Marginal Cost of Public Funds: A Brief Guide[a]

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Abstract

When deciding on the social desirability of public investment, the cost of a project is sometimes adjusted by a factor known as the Marginal Cost of Public Funds ($MCPF$), which captures the cost of raising public funds through distortionary taxation. However, there is no scholarly consensus on its definition or quantification. The purpose of this paper is to provide a brief up-to-date guide to the theoretical background, practical application, and empirical quantification of the $MCPF$, taking into account some recent developments in the public finance literature, and highlighting the broad applicability of the $MCPF$ beyond taxation.

Keywords: benefit-cost analysis, marginal value of public funds, excess burden, distortions, public goods, taxes.

JEL codes: D61, H41, H53, H21

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1 Introduction

What are the trade-offs involved when the government provides a public good? Economic textbooks usually say that the public good should be provided according to the Samuelson rule (Samuelson 1954). This rule says that social welfare is maximized when the public good is provided so that the total amount that people are willing to pay for one more unit is equal to its marginal cost. However, this result assumes that the government can move resources from the private to the public sector at no cost, that is, that the government can use nondistortionary (or "lump-sum") taxes.[1]

At least since Pigou (1928), scholars have discussed how the rule for public good provision should be adjusted to account for distortionary taxation. However, there is still no agreement on how such an adjustment should be made. In this paper, I focus on the adjustment known in the literature as the Marginal Cost of Public Funds ($MCPF$). This welfare measure is widely used by practitioners who, after carefully estimating the effects of a policy, typically make a rough comparison of the benefits to the costs of the program, multiplying the latter by a factor, often thought to be in the range of 1 to 1.5, to capture the economic cost of raising the tax revenue needed to pay for the policy.

The $MCPF$ is often perceived as a confusing concept by researchers and practitioners, and there are many different definitions in the research literature. This is because there are multiple ways of accounting for the behavioral effects of public investment and multiple ways of financing public investment, with different assumptions about what tax instruments are available, how flexible they are, and how they are optimized by the government. Moreover, it is generally not well understood that the $MCPF$ has a broad applicability beyond taxation. In fact, it can be thought of as a general welfare measure that allows empirical researchers estimating causal effects, such as in the context of early childhood education and labor market training programs, to learn about the social desirability of policies, taking into account the long-term effects on tax revenues.

The main purpose of this paper is to provide a brief up-to-date guide to the

[1] This means that the Samuelson rule is a "first-best" result, as opposed to a "second-best" result, where the government has to use distortionary taxes. The early literature on optimal taxation (Ramsey 1927) assumes that lump-sum taxes are not available, without explaining why such taxes are not available. In the modern literature on optimal taxation (Mirrlees 1971), the limits on tax policy come from asymmetric information between the government and taxpayers.

$MCPF$ from a public finance perspective and to discuss some recent developments in the research literature.

In section 2 I begin by introducing the $MCPF$, distinguishing between the "traditional" definition associated with Stiglitz and Dasgupta (1971) and Atkinson and Stern (1974) and the "new" $MCPF$ introduced by Mayshar (1990), which is the main focus of this paper. I also clarify the relationship with the other classical welfare measure, the Marginal Excess Burden (MEB). I then proceed to discuss how empirical studies can be used to quantify the $MCPF$ in section 3. Section 4 discusses the $MCPF$ in the presence of heterogeneous taxpayers and distributional considerations, the $MCPF$ for changes in top tax rates, and the $MVPF$ as introduced by Hendren and Sprung-Keyser (2020). Finally, in section 5 I provide a concluding discussion. Appendix A describes other ways of presenting benefit-cost analysis and relates them to the $MCPF$.

2 The Marginal Cost of Public Funds

2.1 Traditional ways to define the $MCPF$

The focal point in the bulk of the literature on the $MCPF$ has been the following equation (see for example Ballard and Fullerton 1992, page 118):

$$\sum_i MRS^i = MCPF \cdot p. \tag{1}$$

Equation (1) describes that in a social optimum, a public good is supplied such that the economy's total private marginal willingness to pay for an additional unit (as measured by the sum of individuals' marginal rates of substitution between this good and the numeraire consumption good, $\sum_i MRS^i$) is equal to the marginal cost p, adjusted by the Marginal Cost of Public Funds ($MCPF$).

The $MCPF$ in (1) can be divided into three parts. The first part, discussed by Pigou (1928), is the deadweight loss that arises when a distortionary tax is used instead of a lump sum tax and is usually referred to as the Marginal Excess Burden (MEB). In a simple labor supply model, MEB is determined by the compensated labor supply elasticity.[2] The second part captures the fact that

[2]Classical studies that have examined MEB are Harberger (1964, 1974), Browning (1976, 1987), and Hansson (1984). The $MCPF$ has been defined in several different ways in the research literature. Sometimes it is defined as $1+MEB$, but this is different from how $MCPF$

the tax increase leads to an income loss that makes people poorer, leading to income effects on both labor supply and consumption choices. The third part is the effect on individual behavior that results from the public good. The latter two parts were highlighted by Stiglitz and Dasgupta (1971) and Atkinson and Stern (1974) and imply that $MCPF$ can be less than one if, for example, the income effects on the tax base are larger than the substitution effects, so that the overall effect of the tax increase on tax revenues is positive.

MEB reflects a thought experiment in which the tax is raised while each taxpayer receives a hypothetical compensation in the form of a lump sum transfer so that they can achieve the same level of utility as before the tax increase. Instead, $MCPF$ reflects a thought experiment in which the tax increase is used to finance a public good. MEB and $MCPF$ are equivalent if and only if: (i) there are no income effects of the financing tax on labor supply or the demand for private goods, and, (ii) there are no interactions between the public good and demand for private goods or labor supply. Since tax reforms are rarely designed to neutralize income effects, MEB is therefore of limited practical interest.

How should behavioral effects due to the public good (the third part of the $MCPF$) be treated? For example, an infrastructure investment may make work more attractive relative to leisure, thereby reducing the distortion of income taxes on labor supply. It may also increase (or decrease) the demand for taxed private goods and services in a way that increases (or decreases) tax revenue from consumption taxation. There are two main approaches.

The first approach is to include the behavioral effects of the public good in the $MCPF$, which transforms the $MCPF$ into what is commonly referred to as the Social Marginal Cost of Public Funds (SMCF). Variants of SMCF are studied by Wildasin (1984), Mayshar (1991), Snow and Warren (1996), Brent (2006), and Usher (2006). A drawback of the concept is that it is project-specific, which has been discussed by Sandmo (1998).[3]

The second way, which is the most common for tractability reasons, is to assume that public investment does not affect the consumption of goods and services or the supply of labor. A formal way of expressing this is that the

is defined in this paper

[3]An alternative is to include the effects of the public good on the "income side". This means that the $MCPF$ is not affected, but of course requires that the income side is calculated correctly. What is recorded on the income side or the cost side affects what is interpreted as $MCPF$, but is irrelevant to the validity of the policy rule for the public good.

utility function is separable between the public good and both other goods and leisure. Although this is a questionable assumption for many investments, there is often a lack of empirical knowledge about how different public goods interact with demand for goods and labor supply. In light of this, separability may be a useful simplification. This means ignoring, for example, that an investment in infrastructure increases demand for complementary taxed goods, such as vehicles.

Above is the "traditional" way of defining $MCPF$ based on the Samuelson rule, usually referred to as the Atkinson–Stern–Stiglitz–Dasgupta–definition.[4] This definition focuses on the effects of compound budget-neutral reforms, where taxes and spending are adjusted simultaneously. There is often an implicit assumption that public spending is financed by adjusting a proportional tax on labor income. In practice, however, there are many ways to finance a public investment, and each way will produce a different value of $MCPF$. Only in an optimal tax system is $MCPF$ independent of the marginal source of financing.

In the rest of the paper, I will focus on a specific definition of $MCPF$ introduced by Mayshar (1990) and further developed by Slemrod and Yitzhaki (1996, 2001) and Kleven and Kreiner (2006). This definition has been revived by the contributions of Hendren (2016), Finkelstein and Hendren (2020), and Hendren and Sprung-Keyser (2020), whose goal is to popularize the definition among empirical researchers evaluating the impact of public policies.

The alternative "new" definition does not consider budget-neutral composite reforms. Instead, in line with Slemrod and Yitzhaki (2001), two "tax factors" are calculated, $MCPF$, which reflects the marginal cost to society of raising tax revenue to finance a public good, and the Marginal Benefit of the Public Good ($MBPG$), which reflects the marginal benefit to society of spending an additional dollar on a public good G. The decision rule is that spending on the public good should increase as long as $MBPG$ is greater than $MCPF$, and the optimal level is reached when $MBPG = MCPF$. In other words, if it is proposed to increase spending on a public good by $1, the first step is to calculate a tax factor $MBPG$ that describes the welfare effect of spending $1 more on the public good. In a second step, a tax factor $MCPF$ is calculated that reflects the welfare effect of increasing a tax (or decreasing spending on

[4] See Dahlby (2008) for an extensive textbook treatment.

some other project) by \$1.[5]

The main advantage of the new definition over the traditional one is that while the traditional one requires estimates of the elasticity of the tax base in response to combined reforms that change taxes and spending simultaneously (which are difficult to interpret, often project-specific, and rarely estimated in practice), the new definition uses separate estimates of the effects of taxes and government spending on the tax base. This allows causal effects from empirical studies to be used directly without having to be decomposed into income and/or substitution effects (or the effects of government spending), and creates a unified approach that allows researchers to make welfare claims for a wide range of policies beyond taxation, such as early childhood interventions and labor market training programs. The new definition is also more flexible because it allows projects to be financed in arbitrary ways, making it easier to compare different projects and to describe how one project is financed by reducing spending on another. An additional advantage is that the separation in the new definition makes it easier when spending and financing decisions are made at different times or by different branches of government.

2.2 The Mayshar (1990) definition

Mayshar (1990) and Ballard (1990) define $MCPF$ as follows:

$$MCPF = -\frac{\text{Change in welfare in monetary terms}}{\text{Change in tax revenue}}. \tag{2}$$

Expression (2) describes the welfare effect of a project, expressed in dollars, divided by the effect on the government budget. $MCPF$ can be calculated for many types of reforms, not only tax reforms. In case $MCPF$ is calculated for a public project with an expected positive effect on social welfare, it is not so intuitive to describe it as a "cost", therefore we follow Slemrod and Yitzhaki (2001) and define an identical expression as:

$$MBPG = \frac{\text{Change in welfare in monetary terms}}{\text{Change in government expenditure}}. \tag{3}$$

[5] $MBPG$ is a benefit-cost ratio that reflects individuals' private willingness to pay for a project divided by the total cost to the government (including any effects of the project on tax revenues) *without considering the costs of distortionary tax financing*. $MCPF$ and $MBPG$ are formally defined in the next section.

Note that (2) and (3) are mathematically equivalent (one minus the change in tax revenue equals the change in government spending). The reason for presenting two measures is pedagogical. When considering a tax increase, it is intuitive to use expression (2), because a financing tax increase has an expected *negative* effect on welfare and an expected positive change in *tax revenue*. When considering public investment (or a tax cut), it is intuitive to use expression (3) because such a policy has an expected *positive* change in welfare and an expected positive increase in *expenditure*.

Motivated by similar pedagogical reasons, Hendren (2016), Finkelstein and Hendren (2020), and Hendren and Sprung-Keyser (2020) call (2) and (3) the Marginal Value of Public Funds ($MVPF$). $MVPF$ is discussed in more detail in section 4.2.

If the private willingness to pay for a project is 2 dollars, the project costs 1 dollar and increases tax revenues in the long run by 50 cents, then $MBPG = \frac{2}{1-0.5} = 4$. Now consider a tax reform that finances this one dollar cost. Such a tax reform results in a private welfare loss of one dollar, and if it simultaneously reduces tax revenues by 20 cents, then $MCPF = -\frac{-1}{1-0.2} = 1/0.8 = 1.25$. Since $MBPG > MCPF$, implementing the project with the proposed financing implies an increase in social welfare.

2.3 MCPF in a simple labor supply model

Next, I derive an expression for the $MCPF$ for a specific reform, namely a marginal increase in a proportional income tax.[6] I consider a simple labor supply model with no consumption taxes, where leisure is a normal good (i.e., individuals demand more leisure as income increases, ceteris paribus). The economy consists of n identical individuals who each have an hourly wage w and choose their labor supply h so as to maximize individual welfare.[7] The production technology is linear (one hour of work increases the output of the economy by w units) and there is perfect competition.

Since the tax change is small, the welfare effect of the tax change can be approximated by the reduction in disposable income.[8] Before the tax increase,

[6]Later, I consider other policy variations.

[7]Differences across individuals are considered in section 4.

[8]The tax change also affects individuals' labor supply, but since the tax change is small, this behavioral change will have a negligible effect on individuals' welfare. This follows from the envelope theorem in mathematical programming.

each individual had an income of wh and the tax increase of dt therefore implies a reduction in disposable income of $wh \cdot dt$ and a welfare change equal to $-wh \cdot dt$ in monetary terms. Turning to the denominator, the contribution of each individual to government tax revenue is twh and the change in this is $\frac{d(twh)}{dt} \cdot dt$. We can therefore write (2) in the following way:

$$MCPF = -\frac{-wh \cdot dt}{\frac{d(twh)}{dt} \cdot dt} = \frac{wh \cdot dt}{(wh + tw\frac{dh}{dt}) \cdot dt} = \frac{1}{1 + \frac{t}{h}\frac{dh}{dt}} = \frac{1}{1 + \epsilon_{h,t}}, \tag{4}$$

where in the last step we have expressed $MCPF$ in terms of an elasticity. It can be seen that $MCPF$ is a decreasing function of $\epsilon_{h,t}$, the uncompensated elasticity of labor supply with respect to t. Thus, whether $MCPF$ is greater or less than one depends on whether $\epsilon_{h,t}$ is negative or positive. A tax increase distorts labor supply, but at the same time has a positive income effect that increases tax revenues.

$MCPF$ in (4) can also be formally derived from a social optimization problem. Suppose that individuals choose consumption (c) and labor supply (h) in order to maximize their utility $u(c, h, G)$, where utility also depends on the level of a public good G. The budget constraint is given by $c = y + p_w h - p_c c$ where $p_w = (1 - t)w$ is the after-tax wage and y is non-labor income (e.g. wealth or partner income). We normalize the price and tax of consumption to 1, i.e., $p_c = 1$. The indirect utility function $V(p_w, y, G)$ is the value function to the *individual* optimization problem with the following Lagrange function:

$$\mathcal{H} = u(c, h, G) + \lambda(y + p_w h - p_c c), \tag{5}$$

where λ is the shadow price (Lagrange multiplier) of the individual budget constraint. Let $h(p_w, y, G)$ denote the Marshallian demand for h (the uncompensated labor supply function). The government maximizes the welfare of individuals by choosing the tax rate t and the level of public investment G. This results in the following Lagrange function for the government optimization problem:

$$\mathcal{L} = n \cdot V(p_w, y, G) + \mu[n \cdot twh(p_w, y, G) - p_G G], \tag{6}$$

where the marginal production cost of the public good is assumed to be equal to p_G and μ denotes the shadow price (Lagrange multiplier) of the government budget constraint. By exploiting the individuals' Lagrange function (5) to compute $\frac{dV}{dp_w}$ while using the envelope theorem, we obtain that the first-order condition for the government optimization problem with respect to t is:

$$\frac{d\mathcal{L}}{dt} = \frac{dV}{dp_w}\frac{dp_w}{dt} + \mu\left[wh + tw\frac{dh}{dt}\right] = (\lambda h)(-w) + \mu\left[wh + tw\frac{dh}{dt}\right] = 0.$$

If we divide by λhw and rearrange, we get

$$\frac{\mu}{\lambda}\left[1 + \frac{t}{h}\frac{dh}{dt}\right] = 1 \quad \Longleftrightarrow \quad \frac{\mu}{\lambda} = \frac{1}{1+\epsilon_{h,t}}.$$

That is, we have that:

$$MCPF = \frac{\mu}{\lambda}, \tag{7}$$

where μ is interpreted as the social marginal value of public funds and λ as the private marginal value of private funds.

We can alternatively express (2) in terms of the elasticity of taxable income z (where $z = wh$) which is very common to estimate in empirical studies. We see immediately that:

$$MCPF = -\frac{-z \cdot dt}{\left(z - t\frac{dz}{d(1-t)}\right) \cdot dt} = \frac{1}{1 - \frac{(1-t)}{(1-t)}\frac{t}{z}\frac{dz}{d(1-t)}} = \frac{1}{1 - \frac{t}{1-t}\epsilon_{z,1-t}}. \tag{8}$$

Four key observations are in order. First, above I have considered a marginal increase in income tax (which applies to everyone) and the elasticity $\epsilon_{z,1-t} = \frac{1-t}{z}\frac{dz}{d(1-t)}$ should be interpreted as the average elasticity of taxable income in the working population. However, one can consider a tax change only for a particular income group, and then a different measure of $MCPF$ is obtained. As mentioned earlier, only in an optimal tax system is $MCPF$ the same for different sources of marginal finance. In section 4.3 below, I derive $MCPF$ for an increase in the tax on labor income for high-income (top) earners. Second, it should be mentioned that the analysis here assumes "small" tax reforms so that we can ignore the effects of behavioral changes on individuals' utility. This is an important assumption, but at the same time necessary in order to link $MCPF$ to empirically observable elasticities. Third, we have limited the focus of the analysis to "intensive" adjustments (changes in working hours) among individuals who are already working. Kleven and Kreiner (2006) extends the concept of $MCPF$ to account for responses along both the intensive and extensive margins (the decision whether or not to participate in the labor force).[9] The fourth observation, which is perhaps obvious, is that if the aim is to estimate $MCPF$, one does not need to look specifically at empirical studies that have estimated $MCPF$; it is sufficient to start from studies that have estimated relevant elasticities.

2.4 Other tax instruments and financing reforms

In section 2.3, I studied changes in the taxation of labor income. Of course, it is also possible to find expressions for the $MCPF$ for other financing reforms. An important difference between the traditional $MCPF$ and the Mayshar (1990) definition in section 2.2 is that a policy need not be financed in any particular way, and there is a wide range of policies that can be used to close the budget constraint.

One possibility is to close the budget constraint by changing the consumption tax. Under certain assumptions, changes in consumption taxation and income taxation are equivalent, but there are also differences.[10] An important distinction is whether, for example, a public good is financed by a specific commodity tax, such as an increase in the value-added tax on children's toys.[11] Since children's toys are a reasonably small part of an individual's budget, the income effects of such a tax change are small.

Another possibility is to adjust taxes on capital, such as the capital income tax or the corporate income tax. In this case, other models are needed to study the $MCPF$ (taking into account dynamic aspects such as savings behavior). We do not discuss such models here, but note that in such approaches the $MCPF$ would include other elasticities for which we have limited empirical knowledge.

To close the budget constraint, one must not only consider tax policy, but can also think about financing a public project by reducing public spending.

[9] Such responses may be relevant if taxes are raised for low-income individuals, but are not particularly relevant for changes in taxes on high incomes.

[10] See Bastani and Koehne (2022) for an overview of the similarities and differences between labor income taxation and consumption taxation.

[11] It is questionable, however, whether it is a good idea to finance public investment with individual commodity taxes, as this creates distortions in people's consumption patterns, unless there are negative externalities that one wants to counter at the same time (such as excise taxes on alcohol or carbon dioxide emissions).

Hendren and Sprung-Keyser (2020) present over 100 estimates of the $MCPF$ (in their paper, the $MVPF$) for various methods of spending and raising revenue, compiled in their "policy impacts" library for historical policy changes in the United States.[12] They point out the desirability of broadening the empirical goal from thinking only about estimating "the MCPF" to creating a library of estimates that allows researchers to think about raising revenue from different sources.

2.5 The policy rule for the public good

It is instructive to also derive the policy rule for the public good G. By taking the first-order condition with respect to G in (6) we get:

$$\frac{d\mathcal{L}}{dG} = n\frac{dV}{dG} + \mu\left[n \cdot tw\frac{dh}{dG} - p_G\right] = 0.$$

By dividing by λ and rearranging we get:

$$n\frac{dV/dG}{\lambda} = \frac{\mu}{\lambda}\left[p_G - n \cdot tw\frac{dh}{dG}\right]$$

If we denote $MRS^i = \frac{dV/dG}{\lambda}$ and utilize (7), that is, that $MCPF = \frac{\mu}{\lambda}$ we get:

$$\sum_i MRS^i = MCPF \cdot \left[p_G - n \cdot tw\frac{dh}{dG}\right]. \tag{9}$$

Note that in a richer model with different consumption goods and different commodity taxes on them (differentiated commodity taxation), the effects of G on commodity taxes would also appear in (9), see Atkinson and Stern (1974).

The expression (9), which coincides with equation (3) in Atkinson and Stern (1974), illustrates that $MCPF$ according to Mayshar (1990) is identical to the traditional definition presented in (1) if $\frac{dh}{dG} = 0$. A necessary and sufficient condition for this to hold is that the utility function u can be written in the form $u(c, h, G) = u(f(c, h), G)$ for any subutility function f (i.e., the utility function is weakly separable between G and other goods). When the utility function can be written in this form, the marginal rate of substitution between labor and consumption is independent of the public good.

[12] See https://policyimpacts.org/policy-impacts-library.

Note that (9) is a policy rule derived from a simultaneous variation in the income tax and the public good. $MCPF$, as we define it in this paper, does not study such composite (budget-neutral) reforms, but defines separate measures for the public good and the financing tax. Therefore, no separability assumptions are needed to obtain an unambiguous measure of the $MCPF$. A researcher or practitioner studying a public investment policy should construct $MBPG_G = \frac{\sum_i MRS^i}{p_G - n \cdot tw \frac{dh}{dG}}$ of that policy so that it can be compared to all possible ways of raising money for that project (e.g, the $MCPF$ of a revenue-raising tax reform, or the $MBPG$ of some other policy for which spending can be reduced).[13]

3 Empirical quantification

Let us now turn to the empirical quantification of the $MCPF$. Section 3.1 discusses elasticities of taxable income, section 3.2 discusses income effects on labor supply, and section 3.3 discusses implications for the $MCPF$.

3.1 Elasticities of taxable income

Formula (8) and (17) contain the elasticity of taxable income. There is a large empirical literature estimating elasticities of taxable income, and an introduction to this literature is provided by Saez et al. (2012). In general, elasticities of taxable income differ across studies, depending on the nature of the tax reform, the estimation approach used, the country studied, and the income groups affected. A meta-analysis of recent studies is provided by Neisser (2021).[14] For example, based on Swedish, Danish and Finnish tax reforms that affected broad groups of taxpayers, an elasticity of 0.2 could be deemed as reasonable.[15]

In the current context, it is important to note that what enters (8) is the elasticity resulting from a thought experiment where the marginal tax rate is increased in a proportional tax system without compensating households in the

[13]Section 4 and 4.2 discuss how to think about distributional incidence, taking into account that the group of people who benefit from the policy G may be different from those who have to finance it.

[14]See also Aronsson et al. (2022a) for an overview and evaluation of different methods of estimating the elasticity of taxable income.

[15]See Blomquist and Selin (2010) that studied the major tax changes that occurred in Sweden from 1981 to 1991, Kleven and Schultz (2014) that studied the 1987 Danish reform, and Matikka (2018) that studied changes in Finnish municipal taxes from 1995 to 2007.

form of increased (monetary) transfers. Such a reform has a negative substitution effect that is counteracted by a positive income effect (given the reasonable assumption that individuals demand less leisure and more work when income declines). When interpreting elasticities estimated using tax reforms, it thus becomes important to take into account that different tax reforms differ both in terms of which income groups are affected and in the relative importance of income and substitution effects. It is therefore not always straightforward to link estimated elasticities to the simple tax change considered in section 2.3.

A complicating factor is the progressive (non-linear) income tax system. Suppose we are studying high-income earners who are at the beginning of the second segment of a piecewise linear tax schedule, and the tax change under consideration is a lower marginal tax on low incomes combined with an increase in the marginal tax on high incomes. The overall response of high-income earners will reflect both an increase in their marginal tax rate (a substitution effect leading to lower labor supply) and a reduction in the average tax rate due to the tax cut on low incomes (an income effect also leading to lower labor supply). Admittedly, the tax increase on the second segment makes high-income earners poorer (an income effect leading to higher labor supply), but for high-income earners who are just at the beginning of the second segment, this income effect will be negligible.

A conclusion one can draw is that income effects in empirical studies can be both positive and negative depending on whether individuals are poorer or richer overall as a result of the tax form being studied. It is therefore quite possible that studies finding different elasticities are consistent with the same magnitude of substitution effects but different magnitudes of income effects. Unfortunately, few studies are able to shed credible light on the role of income effects (see the discussion in the next section). Many studies therefore ignore the distinction altogether by starting from models where the utility function is linear in consumption, which means that the estimated elasticity is interpreted as a *compensated* elasticity that reflects substitution effects only.

One type of study where income effects tend to play a less significant role is so-called bunching studies (Saez 2010) where elasticities are estimated by locally analyzing behavior around kink points in the tax system (income thresholds where marginal income tax rates discontinuously change).[16] An example

[16]See Kleven (2016) for an overview of bunching studies.

of such a study is Bastani and Selin (2014) who study the first central government income tax kink in Sweden (located in the upper middle part of the income distribution) and find an elasticity of zero for wage earners, which they interpret as an estimate of the *compensated* elasticity. At the same time, the authors point out that if individuals accept a utility loss of not optimizing at the cut-off point of on average one percent of disposable income, the compensated elasticity could be substantially larger.[17]

That elasticities may be underestimated due to optimization frictions does not only apply to bunching studies, but to most empirical studies of how individuals react to taxation. In the labor market, there are several adjustment costs and frictions, for example regarding the possibilities to change one's working hours, change jobs, etc., combined with the fact that it takes time and energy for people to get to know how the tax system works and to understand which tax rates apply.[18] This usually means that: (i) changes in behavior only occur in the longer term, and, (ii) changes only occur if the benefits of changing one's behavior are sufficiently large.[19] However, the vast majority of empirical studies are only able to study responses in the relatively short term. The problem is compounded by the fact that there are responses to taxes that can in principle only be measured in the long run (such as educational choices) and responses that can hardly be measured at all, such as how much effort people put into their workplace in order to get a higher wage, and which are only reflected in labor income after a long time (and which are difficult to attribute to tax changes as income changes over time for many reasons unrelated to taxes).

Finally, while taxes can have a significant negative impact on tax revenues in the long run, *public investment* can also have a significant positive impact on tax revenues in the long run. Examples of this are investments in education that increase labor productivity or investments in health promotion that reduce sickness absence. It is therefore essential to consider the long-term effects of both taxes and public investment. The advantage of the $MCPF$-framework surveyed in this paper is that it is symmetric with respect to the revenue and

[17]In their study, compensated elasticities above 0.39 can be ruled out based on the empirical estimates for 1998 and compensated elasticities beyond 0.7 can only be ruled out based on the estimates for 1999-2005.

[18]Bastani and Waldenström (2021) present recent bunching evidence showing that conditional on income, the responses are larger among those with high cognitive ability.

[19]This is discussed in Chetty (2012), Chetty et al. (2011), Bastani and Selin (2014), Kleven and Schultz (2014), Kostøl and Myhre (2021), and Labanca and Pozzoli (2022), among others

expenditure sides of the government budget.

3.2 Studies of income effects

As discussed in the introduction, a key component of the $MCPF$ for a tax change is the income effects that arise. But how important are they empirically? In the context of the taxable income model studied in section 2.3, the total response to a change in the net-of-tax rate $(1-t)$ can be decomposed using the well-known Slutsky equation as follows:

$$\epsilon_{z,1-t} = \epsilon^c_{z,1-t} + \eta, \tag{10}$$

where $\epsilon^c_{z,1-t}$ is the compensated elasticity of taxable income that describes substitution effects and η is a parameter that captures income effects.[20] The parameter η is defined as:

$$\eta = (1-t)\frac{dz}{dy}, \tag{11}$$

where $\frac{dz}{dy}$ is the marginal propensity to increase one's labor income in response to a marginal increase in non-labor income y. If leisure is a normal good (i.e., the demand for leisure never decreases as income increases), then $\frac{dz}{dy} \leq 0$.

Income effects can be estimated in basically two ways. Either structural labor supply models estimated using data on labor income/hours (z/h), wages (w), taxes (t) and various "other" incomes (y) (such as partner income) are used. One problem with these studies is that they rely on strong assumptions and rarely have access to credible exogenous variation in y. For example, individuals with a strong preference for work relative to leisure will simultaneously work more hours and have more financial assets and therefore greater non-work income, creating a spurious correlation between non-work income and labor supply.

Another way is to use some natural experiment that offers exogenous variation in non-labor income. An important branch of these studies is that which has used lottery winnings. Using lottery winnings offers many advantages over other natural experiments, such as studies based on inheritance (where the ques-

[20]The compensated elasticity is derived from the compensated supply function that describes labor supply adjustments to taxes when individuals are compensated so that they always achieve the same utility level u, see for example Saez (2001) for details.

tion arises to what extent such inheritance is expected or unexpected, and inheritance coincides with the death of a parent which in itself may affect labor supply). One challenge with lottery studies is that they require assumptions about how individuals choose to distribute lottery winnings over the remaining part of the life cycle.

An early study of the effects of lottery winnings on labor supply is Imbens et al. (2001). These authors use data in the United States in the 1980s and find a marginal propensity to increase labor income in response to an income increase of about -0.11, which should be interpreted as an increase in income of 1000 dollars leads to a decrease in labor income of 110 dollars. Cesarini et al. (2017) use Swedish lottery winnings and find a marginal propensity to increase labor income in response to an income increase of between -0.036 (at age 60) to -0.168 (at age 20).[21] This could justify a η of about -0.1.[22]

Golosov et al. (2021) find larger income effects on US data. They find a marginal propensity to increase labor income in response to an income increase of as much as -0.52 (see their Table 4.1), which could easily justify a η of around -0.2. This means that with a value of the compensated elasticity of $\epsilon^c_{z,1-t} = 0.2$, equation (10) yields a value of the uncompensated elasticity that is around zero.[23] Of course, one should be cautious about extrapolating values between countries, as there could also be cross-country differences in compensated elasticities.

3.3 Implications for the $MCPF$

Let us now briefly summarize the implications of the discussion in the two previous subsections. For this purpose, suppose a government agrees on a given value of the elasticity of taxable income for small tax changes that involve broad groups of taxpayers, and let us assume that this value is 0.2. To which extent

[21] See Cesarini et al. (2017), Table 5, Panel C. The authors also report an uncompensated (Marshallian) elasticity of close to zero, 0.009, within their calibrated life-cycle model, see Cesarini et al. (2017), Table 5, Panel D.

[22] Similar results have been found on Dutch data by Picchio et al. (2018) who estimate an average marginal propensity to increase labor income in response to an income increase of -0.056 in the same year that the lottery winnings were received.

[23] This conclusion is consistent with the early studies of labor supply among men that were done in the 1960s, 1970s, and 1980s, see Pencavel (1986) for a review. Early studies found significantly higher elasticities for women (Killingsworth and Heckman 1986) but these elasticities have declined sharply as labor force participation among women has increased, see for example Heim (2007).

should it be interpreted as reflecting income effects?

It is useful to distinguish between two borderline cases. In the first limiting case, 0.2 is interpreted as the uncompensated elasticity of taxable income which implies that $\epsilon_{z,1-t} = 0.2$ and we obtain a value of the $MCPF$ (assuming that $t = 0.5$) of $\frac{1}{1-\epsilon_{z,1-t}} = 1.25$. In the second limiting case, 0.2 is interpreted as the compensated elasticity, which means that we need to add the income effect discussed in section 3.2 according to equation (10) to get the uncompensated elasticity. If we use the estimate $\eta = -0.1$ from Cesarini et al. (2017), we obtain $\epsilon_{z,1-t} = 0.1$, and the $MCPF$ becomes approximately 1.11. This just serves to illustrate how one can go about figuring our reasonable ranges for the $MCPF$ based on empirical estimates.

3.4 The extensive labor supply margin

My presentation of the $MCPF$ has focused on the intensive margin of taxable income. This margin deals with how working individuals' incomes change in response to changes in marginal taxes. The presentation has not explicitly included extensive responses, that is, decisions to work or not to work. Admittedly, elasticities of taxable income reflect the extensive margin to some extent, but the link to $MCPF$ is more complicated because the value of working is controlled by the *average* tax and not the marginal tax.[24] With a positive extensive margin elasticity, the tax factor is greater than one even if the uncompensated elasticity of taxable income is zero. However, participation elasticities are highly context-dependent as they are determined by how many people in the labor force are indifferent on the margin between working and not working and whose decision to work is affected by a small change in the average tax rate induced by a small change in the marginal tax rate.[25] Another drawback with participation elasticities is that they only consider the voluntary part of the participation decision, neglecting the fact that many who would like to work cannot find work due to labor demand considerations such as minimum wages

[24] Kleven and Kreiner (2006) develop $MCPF$ in a context of both intensive and extensive margins of labor supply.

[25] Bastani et al. (2021) is a recent study that presents quasi-experimental evidence on labor supply responses along the extensive margin in response to changes participation tax rates, exploiting a reform of the housing allowance in Sweden in the late 1990s. They find an average participation elasticity of around 0.13 for their study population of married women with relatively low income levels. They also show that the elasticities decline with income.

in combination with insufficient skills.

4 Distributional considerations

The presentation so far has focused on proportional changes in income taxes affecting all taxpayers, who have been assumed to be identical. We now discuss the implications of relaxing this assumption. Section 4.1 presents the most common way to define the $MCPF$ in the presence of heterogeneous taxpayers and distributional concerns in the prior literature. Section 4.2 discusses the way distributional concerns are handled in the context of the marginal value of public funds as presented by Hendren and Sprung-Keyser (2020). Section 4.3 derives the $MCPF$ for a tax increase on high income earners using a perturbation argument as in Saez (2001).

4.1 The MCPF and distributional considerations

One limitation of the $MCPF$ as I have presented it above is that it focuses only on the distortionary costs of taxation without considering the distributional effects. This is problematic because the reason the government uses distortionary taxation is to redistribute income. If the distribution of income did not matter, the government could use a non-distortionary lump sum tax, which would mean that public investment could be financed without distortion and the $MCPF$ would be obsolete.

It is not clear how $MCPF$ should be generalized to consider distributional aspects. The most common variant, defined by Johansson-Stenman (2005), Gahvari (2006), and Kleven and Kreiner (2006), is a generalization of the definition in (7) as follows:

$$MCPF^{\text{dist}} = \frac{\mu}{\sum \pi^i \lambda^i}, \tag{12}$$

where I have added the superscript "dist" to distinguish this definition from the one with a representative agent in (7).[26] In the numerator we have the marginal social value of public funds, and in the denominator we have the marginal utility λ^i of different individuals in the economy, weighted by each individual's importance in the social welfare function, π^i.

[26]This notation will be omitted where not necessary below.

The marginal social value of public funds μ is derived from a social optimization problem that includes distributional considerations. Let $W = \sum_i \pi^i U^i$ denote social welfare, where U^i is the utility of individual i, and let R denote net tax revenue (taxes minus expenditures). Then consider a small change in policy, captured by the parameter z (reflecting, for example, a change in public spending on a project or a change in the tax-transfer system). Taking the first-order condition of the Lagrangian expression $\mathcal{L} = W + \mu R$ w.r.t z yields:

$$\mu = -\frac{\frac{dW}{dz}}{\frac{dR}{dz}} = -\frac{\sum_i \pi^i \frac{dU^i}{dz}}{\frac{dR}{dz}} = -\frac{\sum_i \pi^i \lambda^i \left(\frac{dU^i}{dz} \Big/ \lambda^i\right)}{\frac{dR}{dz}} = -\frac{\sum_i \pi^i \lambda^i WTP_z^i}{\frac{dR}{dz}}, \quad (13)$$

where $WTP_z^i = \frac{dU^i}{dz} \Big/ \lambda^i$. Thus, we have that:

$$MCPF^{\text{dist}} = -\frac{\sum_i \pi^i \lambda^i WTP_z^i}{\frac{dR}{dz} \cdot \left(\sum_i \pi^i \lambda^i\right)}. \quad (14)$$

It is clear that $MCPF^{\text{dist}}$ depends on the extent to which taxes and transfers redistribute between individuals (and what tax instruments are available), since the degree of redistribution affects λ^i. It is also clear that the measure depends on the government's preferences for redistribution π^i.[27] Only when the government has access to *individualized* lump-sum taxation does $MCPF^{\text{dist}}$ collapse to $MCPF$ in (7). However, individualized lump-sum taxation requires the government to observe each individual's underlying ability to earn income, for which, not surprisingly, methods are lacking. It is also clear that the willingness to pay WTP_z^i for a given policy will generally differ across individuals. For example, the proportional income tax change in section 2.3 would imply a larger tax burden for people with higher w (see equation 4).

Common to studies based on $MCPF^{\text{dist}}$ as defined in (12) is that the welfare measure depends on distributional considerations that are affected by policymakers' preferences for redistribution, the tax instruments available, and the extent to which tax instruments can be used to achieve redistributive goals (e.g., there are political constraints that limit how taxes can be adjusted in practice). Three cases can be distinguished:

[27] The interpretation of $MCPF$ in models with distributional aspects and non-linear taxation of income has been discussed by Christiansen (1999).

1. If the current tax system is optimal, the efficiency cost of a small tax increase will exactly match the distributional gains.

2. If the current tax system is less redistributive than what the policymaker considers optimal, a small tax increase to finance a public good will have a distributional gain that exceeds the efficiency cost.

3. If the current system is more redistributive than what the decision-maker considers optimal, a small tax increase will have a distributional cost (the redistributive efficiency of the tax system moves even further from the decision-maker's optimum), which is added on top of the efficiency cost of the financing tax change.

A large literature in public finance has analyzed optimal provision of public goods under the assumption that the government redistributes among individuals with different abilities to earn income using an optimal nonlinear income tax.[28] Christiansen (1981) and Boadway and Keen (1993) show that in this case the policy rule for a public good is the same as in a first-best setting (the Samuelson rule) without any adjustment for the cost of raising tax revenue. The result is based on a model in which preferences for labor supply are separable from other goods, including the public good.[29] One way to understand this result is that the nonlinear income tax $T(z)$ includes a lump-sum transfer $T(0)$ that can be reduced to finance the public good at no efficiency cost. Such an adjustment has distributional effects, but these can be neutralized by adjustments in the nonlinear income tax so that all individuals achieve the same welfare as before.[30]

It is tempting to interpret it as $MCPF = 1$ under optimal nonlinear taxation. However, Gahvari (2006) shows that it is actually less than one in the model of Boadway and Keen (1993).[31] In a more general model, Gahvari (2006) shows

[28]This is the starting point of modern tax research, which assumes that the fundamental constraint on tax policy is asymmetric information about individuals' abilities, see Mirrlees (1971).

[29]If preferences are not separable, a "modified" Samuelson rule applies instead, which takes into account the effects of the public good on income redistribution (through the so-called self-selection constraints) as well as the tax revenue from commodity taxes (see, for example, Edwards et al. 1994 and Aronsson et al. 2022b). While these effects depend on the tax wedge, they are not very meaningful to relate to the $MCPF$.

[30]Kaplow (1996, 2004) argues that the first-best Samuelson rule is relevant even if the tax system is not optimal, as long as the introduction of the public good and its financing can be done in a distributionally neutral way.

[31]We thus have that $MCPF$ is less than one even though the public good in the optimum is

that it can actually be both less than and greater than one. Thus, there is no "definitive" value of $MCPF$ in models of nonlinear income taxation.

Some research has studied $MCPF$ in the presence of distributional aspects under restricted tax systems. Sandmo (1998) studies the policy rule for public goods under an optimal *linear* income tax (proportional taxation of labor income combined with a uniform lump-sum transfer) and shows that the $MCPF$ in this case is less than one. The reason is that the public good can be financed at the margin without efficiency cost by reducing the lump-sum transfer. As this reduction makes people poorer, tax revenues increase through income effects while distributional effects are zero since the tax system is assumed to be optimal from the outset.[32]

Jacobs (2018) builds on Sandmo (1998) and proposes a modified measure of $MCPF$ based on Diamond (1975). With this measure, the income effects of tax financing are included in the social value of private funds (see also Lundholm 2005). With this definition, $MCPF = 1$ under both the optimal linear tax system and under an optimal non-linear income tax. However, this modified definition has not yet gained traction in the research literature.[33]

4.2 The Marginal Value of Public Funds (MVPF)

Hendren and Sprung-Keyser (2020) define $MCPF$ in the presence of distributional concerns in a slightly different way than above. They retain the original Mayshar (1990) definition of the $MCPF$ and define the Marginal Value of Public Funds (MVPF) as the total willingness to pay of affected individuals $\sum_i WTP^i$ divided by the net cost of the policy:

$$MVPF = \frac{\sum_i WTP_z^i}{-\frac{dR}{dz}}. \quad (15)$$

provided neither "under" nor "above" the Samuelson rule. Note, however, that the level of the public good can be both higher and lower in second-best compared to first-best.

[32]The $MCPF$ according to the definition in Sandmo (1998) is the same whether the marginal financing is done through a reduction in lump sum transfer or through the distortionary income tax rate. However, the lack of flexibility in the income tax (due to the linear rather than non-linear nature of the income tax) gives rise to a distribution factor linked to the public good in the policy rule, but this is included on the "revenue" side and not on the cost side. With optimal non-linear taxation, this generally does not arise because distributional issues can be dealt with entirely by income taxation (under certain separability assumptions)

[33]See Bos et al. (2019) for a discussion.

Thus, except for the normalization by the average private marginal utility of income ($\sum_i \pi^i \lambda^i$), the main difference between $MCPF^{\text{dist}}$ in (14) and $MVPF$ in (15) is that the willingness to pay in the numerator of (15) does not include the distributional weights $\pi^i \lambda^i$. This has the benefit that the $MVPF$ is computed without taking a position on the social welfare weights.

To evaluate policy proposals that benefit one group and are paid for by another, note that (13) can be rewritten as follows:

$$\mu = -\frac{\sum_i \pi^i \lambda^i WTP_z^i}{\frac{dR}{dz}} \cdot \frac{\sum_i WTP_z^i}{\sum_i WTP_z^i} = -\sum_i \pi^i \lambda^i \frac{WTP_z^i}{\sum_i WTP_z^i} \cdot \frac{\sum_i WTP_z^i}{\frac{dR}{dz}}$$
$$= \eta \cdot MVPF, \tag{16}$$

where $\eta = \sum_i \pi^i \lambda^i \frac{WTP_z^i}{\sum_i WTP_z^i}$ is the average social marginal utility of income weighted by the *economic incidence* $\frac{WTP_z^i}{\sum_i WTP_z^i}$ of policy z.

Equation (16) shows that the (unweighted) $MVPF$ can be multiplied by η to convert from units of recipient income (i.e., the willingness to pay of those targeted by the policy) to units of social welfare. For example, suppose the $MVPF$ for changing the top tax rate is 1.85 and the $MVPF$ for expanding the Earned Income Tax Credit for families with children (EITC) is 1.15. This means that the government should spend more on the EITC financed by higher top tax rates if it values \$1.15 for the poor more than \$1.85 for the rich, i.e., if $\eta_{\text{poor}} \cdot 1.15 > \eta_{\text{rich}} \cdot 1.85$.

Conversely, if we observe that a government expands the EITC financed by increases in top tax rates, we can infer that $\frac{\eta_{\text{poor}}}{\eta_{\text{rich}}} > \frac{1.85}{1.15}$. In this way, the $MCPF/MVPF$ framework is related to the "inverse optimal tax" literature which attempts to derive the social weights that would rationalize the current tax schedule as optimal.[34] In a social optimum, $\frac{MVPF_{z_A}}{MVPF_{z_B}} = \frac{\eta_{z_A}}{\eta_{z_B}}$ for any policies z_A and z_B targeting income groups A and B, respectively. Thus, the $MVPF$ allows capturing the key insight of the Mirrlees (1971) framework that redistribution is costly and that the costs of redistribution differ along the income distribution.

[34] For contributions to the inverse optimal tax literature, see, for example, Christiansen and Jansen (1978), Bourguignon and Spadaro (2012), Bargain et al. (2014), Lockwood and Weinzierl (2016), Jacobs et al. (2017), Bastani and Lundberg (2017), and Hendren (2020).

4.3 MCPF for an increase in the top income tax rate

In section 2.3 we calculated $MCPF$ for proportional tax change in an economy of identical individuals. Suppose now that agents are heterogeneous in terms of their income and we increase the marginal tax rate by dt only above a certain income level $\bar{z}$. We assume that in the initial situation everyone faces the same tax rate t so that the result of the reform is a piece-wise linear tax schedule where taxpayers face tax rate t up to the income level $\bar{z}$ and face tax rate $t + dt$ above that (for $z > \bar{z}$). Such a reform has exactly the same effects on individuals with incomes $z \geq \bar{z}$ as a two-part reform with two components: (i) a marginal tax increase of dt on incomes from $z = 0$ to $z = \infty$, and, (ii) a lump-sum compensation with size $\bar{z}dt$. The second component is necessary because a tax increase that covers only a portion of income does not make individuals as much poorer as a tax increase that covers all income. Saez (2001) shows how the income change to this reform for an individual with initial income z can be written as

$$dz = \frac{\partial z}{\partial(1-t)}dt + \frac{\partial z}{\partial y}\bar{z}dt = -(\epsilon_{z,1-t}z - \eta\bar{z})\frac{dt}{1-t},$$

and that the total reduction in tax revenue can be written (where $\mathbf{E}_{z>\bar{z}}$ means that we take an average over all individuals with income higher than $\bar{z}$)

$$\mathbf{E}_{z>\bar{z}}[t \cdot dz] = -t \cdot (\bar{\epsilon}_{1-t}z_m - \bar{\eta}\bar{z})\frac{dt}{1-t},$$

where z_m is the average income, $\bar{\epsilon}_{1-t}$ is the average uncompensated elasticity, and $\bar{\eta}$ is the average income effect for individuals with incomes higher than $\bar{z}$. We can use this to derive an expression equivalent to (8) but which applies to a tax increase dt only for individuals with incomes above $\bar{z}$:

$$MCPF^{\text{top}} = \frac{(z_m - \bar{z}) \cdot dt}{(z_m - \bar{z}) \cdot dt - t \cdot (\bar{\epsilon}_{1-t}z_m - \bar{\eta}\bar{z})\frac{dt}{1-t}} = \frac{1}{1 - \frac{t}{1-t} \cdot (\bar{\epsilon}_{1-t} \cdot a - \bar{\eta} \cdot b)}, \tag{17}$$

where $a = \frac{z_m}{z_m - \bar{z}}$ is the so-called "Pareto parameter" which is a measure of how "thin" the distribution of high incomes is above a certain level $\bar{z}$ (which is the level of income above which the tax is raised) and $b = \frac{\bar{z}}{z_m - \bar{z}}$ reflects how much of the total income is not subject to the tax increase (how much of the income is infra-marginal to the tax increase). Note that if $\bar{z} = 0$ so that the tax reform covers all income, $a = 1$ and $b = 0$ which means that (17) becomes identical to (8).[35] Bastani and Lundberg (2017) study the distribution of income in Sweden locally over a limit $\bar{z} = 3 \cdot z_{avg}$ where z_{avg} is the average labor income in the economy. They find that a ranged between 3 and 4 over the period 1971-2012. If we set $a = 3$, it necessarily follows that $z_m = 4.5 \cdot z_{avg}$. This in turn implies that $b = \frac{3 \cdot z_{avg}}{4.5 \cdot z_{avg} - 3 \cdot z_{avg}} = 2$. If we assume $\bar{\epsilon}_{1-t} = 0.2$, $t = 0.5$ and $\eta = 0.2$, we obtain $MCPF^{\text{top}} = \frac{1}{1-(0.2 \cdot 3 - 0.2 \cdot 2)} = 1.25$.[36]

5 Concluding discussion

This paper has discussed how to account for the welfare losses that arise when public investment is financed by distortionary taxes, incorporating some recent developments in the public finance literature. My presentation of these costs has been based mainly on the definition of the Marginal Cost of Public Funds ($MCPF$) introduced by Mayshar (1990), Ballard (1990), Slemrod and Yitzhaki (2001), and recently received new attention in the form of the Marginal Value of Public Funds ($MVPF$) by Hendren (2016), Hendren and Sprung-Keyser (2020), and Finkelstein and Hendren (2020).

If a public project is to be financed by a small tax change, the $MCPF$ for the financing tax reform can be expressed in terms of elasticities estimated in the large empirical research literature that has studied how individuals respond to changes in taxes and transfers. Thus, those interested in the magnitude of the $MCPF$ for a tax change need not limit themselves to studies that explicitly calculate the $MCPF$, but can learn from a wide range of studies that have used different strategies for identification and estimation.

Different types of tax reform have different $MCPF$. If a project is financed by a proportional increase in the marginal tax rate on labor income that covers

[35] Saez et al. (2012), page 8, calculate $MCPF$ for a tax increase on top incomes without taking into account income effects and finds that $MCPF^{\text{top}} = \frac{1}{1-\frac{t}{1-t} \cdot a \cdot e}$ where e is the compensated elasticity of taxable income. We get exactly the same expression if we put $\eta = 0$ and $\bar{\epsilon}_{1-t} = e$ in equation (17).

[36] Miao et al. (2022) study the phasing out of the Swedish working tax credit that was implemented in Sweden in 2016 (an increase in the marginal tax rate on top income earners) and find elasticities of between 0.13 and 0.16 over a three-year period for individuals above the 95th percentile of labor income.

all income groups, a very simple expression can be derived. This expression is equal to $\frac{1}{1-\frac{t}{1-t}\epsilon_{z,1-t}}$, where $\epsilon_{z,1-t}$ is the *uncompensated* elasticity of taxable income with respect to the net-of-tax rate (one minus the tax rate) and t is the current tax rate level.

The reason why the uncompensated elasticity is relevant in calculating the $MCPF$ for a tax reform that finances public investment (e.g. tax-financed infrastructure) is that such tax reforms involve a loss of income for households. This income effect means that although the tax increase at the margin makes it less profitable to work, people become poorer and therefore have incentives to work more to maintain their consumption level. Thus, armed with an elasticity of taxable income, one must assess the extent to which this elasticity captures these income effects. If it doesn't, an external estimate of income effects can be used, for example from recent studies examining behavioral responses to lottery winnings.

Larger investments or the combination of many small projects (e.g., in the context of an infrastructure bill) require larger tax increases, and the $MCPF$ may be larger than most current estimates of tax elasticities would imply. This is because labor market decisions (e.g., decisions to change jobs or reduce hours) are often discrete-that is, they are made only when the value of changing behavior is sufficiently large relative to the cost-and empirical studies have found that taxpayers tend to respond more to large tax changes than to small ones (see, e.g., Kleven and Schultz 2014). There is also uncertainty about the magnitude of elasticities in the long run because most empirical studies have a relatively short-term perspective and some margins are difficult to measure at all, such as effects on educational choices and career aspirations.

It is well understood that because individuals respond to taxation, the actual increase in tax revenue from a tax increase can be both lower and higher than the purely mechanical increase in revenue. What is perhaps less well understood, however, is that behavioral effects of a public investment can cause the actual cost of a project to differ from the mechanical cost. For example, improved infrastructure may lead to new jobs, increased accessibility, reduced travel time, and reduced risk of accidents, air pollution, and noise, all of which lead to changes in income that, in the long run, contribute to increased tax revenues that fully or partially offset the mechanical costs. These indirect social benefits are called fiscal externalities because they are not included in individuals' private willingness to pay.[37]

An important message of the paper is that the $MCPF$ should be calculated for both tax and spending policies. The $MCPF$ for a tax increase captures how the tax change reduces individuals' disposable income, leads to a mechanical increase in tax revenues, and affects tax revenues through effects on individuals' behavior. The $MCPF$ for a public spending project captures individuals' private willingness to pay for the project, the mechanical cost of the project, and the impact of the project on tax revenues through effects on individuals' behavior. Thus, the relevant welfare measure is not different when spending and tax policies are considered, and tax and spending policies should be given equal weight in assessing the long-term consequences for the government budget.[38]

A few final comments are in order. First, the gross costs to which the $MCPF$ are applied to are easier to estimate ex-post than ex-ante. Thus, there is a difference between the calculation made by a researcher evaluating past projects and that made by a practitioner deriving benefit-cost rules for current projects. Second, there are many costs of public investment that are not typically captured in the $MCPF$ framework, such as crowding-out of private investment, distortions of market competition and inefficient public procurement practices. Third, throughout this paper I have referred to the "new" or "alternative" $MCPF$ as opposed to the traditional $MCPF$ approach introduced by Atkinson and Stern (1974) and others. Perhaps it is time, as suggested by Hendren and Sprung-Keyser (2020), to rename the $MCPF$ as the $MVPF$ to avoid the confusion that has plagued the earlier literature and to emphasize the broad applicability of the welfare measure beyond taxation.

[37]An investment in public transportation may, for example, reduce the risk that a key person in a workplace will be late for work. This has an obvious value for the individual, but also additional positive effects for society (effects for the company, the employees, etc.) that the individual does not include in his own utility calculation.

[38]Traditionally, textbooks teach empirical researchers to conduct a cost-benefit analysis and "adjust for the MCPF," but this has often resulted in including the effects of tax increases while ignoring the positive effects of spending on the government budget. For example, Heckman et al. (2010) evaluates the return to the HighScope Perry Preschool Project and adjusts for the welfare costs of increased tax revenues, but not for the positive effects on the government budget of the increased future earnings of participating children.

A Net Social Benefit and Benefit-Cost Ratios

It is instructive to briefly outline other ways of performing benefit-cost analysis and relate this to the $MCPF$. A classical way of evaluating projects, at least since Feldstein (1964), is to calculate the net benefits of a project. García and Heckman (2022a) define Net Social Benefit (NSB) in the following way (see also García and Heckman 2022b):

$$NSB = B - D(1+\phi) + \Omega(1+\phi), \tag{A1}$$

where B is the direct welfare effect, D is the direct cost, and Ω is the benefit to society at large and $\phi = MEB$. Here, Ω may capture, for example, that part of the project cost is recovered in the long run through cost savings. The multiplication by $1+MEB$ is justified by the fact that one dollar in the hands of the government is valued at $1+MEB$ because the marginal source of financing has a welfare loss amounting to MEB.[39]

An advantage of calculating net social benefits over benefit-cost ratios is that the former concept takes into account the scale of social benefits and avoids the arbitrariness of what to include in the denominator or numerator. One problem, however, is that large projects tend to be ranked highest and the measure is sensitive to project delineation (lumping together two projects with positive but low NSB results in a new project with higher NSB).

When calculating NSB, it is also common to calculate the net benefit *per dollar* (NBD):

$$NBD = \frac{NSB}{D} = \frac{B}{D} - (1+\phi) + \frac{\Omega}{D}(1+\phi). \tag{A2}$$

If not all profitable projects can be implemented, it is important to look at both NSB and NBD. To see this, assume that the project benefit can be described as $B = (1+\phi)D + \gamma$ for $\gamma \geq 0$ and that $\Omega = 0$. This means that $NSB = \gamma$ and $NBD = \frac{\gamma}{D}$. Suppose we have two types of projects, a large project with $D = 100$ and $\gamma = 1000$, and a smaller project with $D = 8$ and $\gamma = 100$.

[39] Remember that MEB reflects a different thought experiment than $MCPF$ and that $1 + MEB = MCPF$ only if the financing tax increase has no income effects on individuals' behavior. For example, it is always true that $1 + MEB \geq 1$ but $MCPF$ can be less than one or even negative if the income effects are large enough.

The large project has $NSB = 1000$ and $NBD = 10$. The smaller project has $NSB = 100$ and $NBD = 12.5$. The smaller project thus has lower NSB but higher NBD. Since the smaller project has a higher NBD, it means that if we have a budget of 100, and can implement 12 projects of the smaller project type, we get a total NSB of 1200 which is higher NSB than the large project.

How does NSB relate to the classical benefit-cost ratio (BCR) defined in, for example, Boardman et al. (2018)? Using the notation above, BCR becomes the following:

$$BCR = \frac{B + \Omega(1+\phi)}{D(1+\phi)}. \tag{A3}$$

The BCR quotient is thus created by "moving" the cost $D(1+\phi)$ to the denominator. Note that $NSB > 0$ if and only if $BCR > 1$. Therefore, exactly the same projects are judged to be socially desirable under NSB and BCR. However, the choice of metric affects the distance between projects, which matters if not all projects with positive net benefits can be implemented.

If we also "move" $\Omega(1+\phi)$ to the denominator (in the form of a reduced cost), we get:

$$BCR' = \frac{B}{D(1+\phi) - \Omega(1+\phi)}. \tag{A4}$$

Again, this manipulation does not affect which projects are deemed profitable, but does affect the ranking. If we assume that we have D dollars to spend, and avoid making an assumption about how D is financed, we can set $\phi = 0$ and get:

$$BCR'' = \frac{B}{D - \Omega}. \tag{A5}$$

The above expression is equal to $MCPF$ if we restrict Ω to represent the long-term behavioral effects of the project on *tax revenue* (and allow other positive welfare effects to be included in B). For example, we see that if $\Omega \to D$ (the project almost pays for itself) then $BCR'' \to \infty$ holds regardless of the size of $B > 0$. In future studies, numerical calculations illustrating the ranking of projects under different definitions of benefit-cost criteria would be useful to understand the practical significance of different assumptions.

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